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Overall Separation Factor in a Gas Centrifuge Using a Purely Axial Flow Model

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Abstract: A unit molar weight mass difference diffusion equation was derived and solved by the radial averaging approximation method for binary component isotopes. There was no limit to the range of concentration value. The expression of γ_0 was given for arbitrary concentrations. In addition, the analytical solution of a purely axial flow in a gas centrifuge was obtained by a linearized motion equation for a centrifuge. The solution was not under the condition that the speed parameter $A^2 \gg 1$.

Calculation examples show the distributions of different magnitudes of A^2. The parameters that influence the value of γ_0, such as the feed flow rate, F, the product, ρD, the speed parameter, A^2 and the wall pressure, p_w, are discussed.

Keywords: Gas centrifuge, overall separation factor, concentration equation, purely axial flow

INTRODUCTION

The overall separation factor for the unit molar weight difference, γ_0, presented by Chuntong Ying and Zhixing Guo (1), is an important factor to describe the separation characteristics of a gas centrifuge. The very valuable relationship between the separation factors and mass difference is $\gamma_{ij} = \gamma_0^{M_j - M_i}$, where γ_{ij} is the separation factor of a gas centrifuge between the ith and the jth isotopes and M_i is the molar weight of the ith component,

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M_j is the molar weight of the jth component. The separation factor γ_0 depends on variables such as the flow pattern, size, operating parameters and physical properties. Chuntong Ying and Wood et al. (2, 3) also studied variables that influence γ_0 and made use of the concept of the separation power of binary isotopes to estimate γ_0 in a gas centrifuge. The dependence of flow field on parameter A^2 is explicitly provided in the analytical expression of flow field in current paper and the calculation process is also much simpler and more convenient than the reported papers (2, 3). In addition, the overall separation factor is derived by solving the concentration equation for any concentration. Therefore the calculation result of the overall factor is more exact than the estimation value obtained in the reported papers.

The distribution of the flow field must be known in order for the diffusion equation to be solved, and thus a simplified mathematical model, called the purely axial flow model, was established. Some scholars have investigated this model. Berman (4) considered the radial direction variation of temperature as the drive producing countercurrent flow in a centrifuge. The temperature was a function of the radial coordinate only. Using this assumption Berman simplified the equation of motion and obtained an integral equation of the axial velocity. The profiles of the axial velocity, temperature and pressure were obtained after enormous numerical integration. Unlike Berman, Soubbaramayer (5) treated the axial direction variation of temperature as the countercurrent flow drive. A three-order differential equation of the axial velocity was obtained after the equations of motion were linearized. But no explicit expression was given for the axial velocity. Lotz (6) also provided the axial temperature gradient as the countercurrent flow drive. After the linear set of conservation equations was simplified, an integral equation of the axial velocity was solved by a numerical method. Similar to Soubbaramayer, Von Halle (7) provided a special supposition, which was called the thin layer approximation, according to the characteristics of a high peripheral speed centrifuge separating larger molecular weight uranium isotopes. Based on the approximation, analytical solutions of the internal flow were obtained. It is convenient to analyze separation performance with a larger value of speed parameter A^2 ($A^2 \equiv M(\Omega\, r_a)^2/(2RT_0)$).

In this paper, the diffusion equation of unit molar weight mass difference is solved by a radial averaging approximation method for a binary isotope mixture. There is no limit to the range of concentration value that can be used to solve the diffusion equation. The solved expression of γ_0 can be applied to an arbitrary concentration. The analytical solution of the purely axial flow model in a gas centrifuge, which is not under $A^2 \gg 1$ conditions, is derived by reducing the linearized motion equations of the thermal countercurrent flow for centrifuges in order to solve the diffusion equation.

Calculation examples illustrate the influence of some parameters, such as the feed flow rate, F, the product, ρD, the speed parameter, A^2 and the wall pressure, p_w, on γ_0. The distributions of the axial velocity, the axial mass

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flux and the flow pattern efficiency for different magnitudes of speed parameter A^2 are also shown.

SOLUTION OF THE UNIT MASS DIFFERENCE DIFFUSION EQUATION

A definition of the separation factors of a gas centrifuge between the ith and the jth isotopes, γ_{ij}, has been given as (1)

$$\gamma_{ij} \equiv \frac{C_{iP}/C_{jP}}{C_{iW}/C_{jW}} \tag{2.1}$$

In terms of paper (1), γ_{ij} and γ_0 have the following relationship

$$\gamma_{ij} = \gamma_0^{M_j - M_i} \quad i = 1, \ldots, n; \;\; j = 1, \ldots, n \tag{2.2}$$

where M_i is the molar weight of the ith component, M_j is the molar weight of the jth component, and γ_0 is the overall separation factor per unit molar weight difference. From the definition of the separation factors, it is first necessary to solve a set of concentration equations to get the product and waste concentrations, and then finally to obtain the separation factors. If the isotope mixture is considered as a binary component and its molar weight difference is unit mass difference, the overall separation factor of the binary isotope mixture is γ_0.

The concentration equation in a single gas centrifuge is (1)

$$\left(\frac{1}{2\pi\rho D_i}\int_0^{r_a}\frac{\psi^2}{r}dr + \pi\rho D_i r_a^2\right)\frac{dC_i}{dz} = \frac{\Omega^2(\overline{M} - M_i)C_i}{RT} \times \int_0^{r_a}\psi r dr - (P^*_{iL} - P^* C_i) \quad i = 1, 2, \ldots, n \tag{2.3}$$

where C_i is the radial averaged concentration of the ith isotope in a multicomponent mixture with n components; ψ is the stream function, $\psi = \int_0^r \rho V_z \pi r dr$, V_z is the axial component of the velocity of the mixture in the gas centrifuge, P^* is the net axial flow flux of the mixture, P^*_{iL} is the net axial flow flux of the ith component, ρ is the density of process gas, D_i is defined as $D_i = \sum_{k=1}^{n} (C_i/D_{ik})^{-1}$, and D_{ik} is the binary diffusion coefficient. $\overline{M}$ is the average molecular weight of the mixture, that is, $\overline{M} = \sum_{i=1}^{n} M_i C_i$. Now considering the binary mixture, let C be the light component concentration. Then

$$\overline{M} - M_i = \sum_{j=1}^{2} M_j C_j - M_i = \Delta M(1 - C) \tag{2.4}$$

where ΔM is the molar weight mass difference of the binary isotope mixture. In the system of a binary isotope mixture, the diffusion Eq. (2.3) is

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written as

$$\left(\frac{1}{2\pi\rho D}\int_0^{r_a}\frac{\psi^2}{r}dr + \pi\rho D r_a^2\right)\frac{dC}{dz} = \frac{\Delta M\Omega^2 C(1-C)}{RT} \times \int_0^{r_a}\psi r dr - P^*\left(\frac{P_L^*}{P^*} - C\right) \tag{2.5}$$

For the reduction expression, the following dimensionless parameters are defined

$$\eta = \frac{z}{r_a}; \quad \eta_H = \frac{z_H}{r_a}; \quad \eta_F = \frac{z_F}{r_a}; \quad \varphi = \frac{P^*}{\pi\rho D r_a}; \quad \varepsilon_{s0} = \frac{\Omega^2 r_a^2}{2RT};$$
$$E_F = \frac{4(\int_0^{r_a}\psi r\,dr)^2}{r_a^4\int_0^{r_a}(\psi^2/r)\,dr}; \quad m = \frac{(\int_0^{r_a}(\psi^2/r)\,dr)^{1/2}}{\sqrt{2}\pi\rho D r_a} \tag{2.6}$$

where, z_F is the axial location of the feed point and z_H is the axial length of the gas centrifuge. Suppose $\Delta M = 1$, substituting Eqs. (2.6) into (2.5) result in

$$(m^2+1)\frac{dC}{d\eta} = \varepsilon_{s0}\sqrt{2E_F}mC(1-C) - \varphi\left(\frac{P_L^*}{P^*} - C\right) \tag{2.7}$$

The above differential equation can be rewritten by

$$-\frac{dC}{d\eta} = k_0(C-a)(C-b) \tag{2.8}$$

where

$$k_0 = \frac{\varepsilon_{s0}\sqrt{2E_F}m}{m^2+1}$$
$$a = \frac{1}{2}\left[(1+x) - \sqrt{(1+x)^2 - 4\frac{P_L^*}{P^*}x}\right]$$
$$b = \frac{1}{2}\left[(1+x) + \sqrt{(1+x)^2 - 4\frac{P_L^*}{P^*}x}\right]$$
$$x = \frac{\varphi}{\varepsilon_{s0}\sqrt{2E_F}m}$$

Here k_0, a, b, and x have different values in the enriching and stripping sections. With the boundary conditions given, the head and tail concentration can be obtained by solving Eq. (2.8).

The boundary conditions are

$$\text{for the enriching section,} \quad \begin{matrix} \eta = \eta_F & C = C_0 \\ \eta = \eta_H & C = C_P \end{matrix} \tag{2.9}$$

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$$\text{and for the stripping section,}\quad \begin{array}{ll} \eta = \eta_F & C = C_0 \\ \eta = 0 & C = C_W \end{array} \tag{2.10}$$

The conditions are given based on the assumption that the feed is injected into the internal axial flow inside the centrifuge. Here, C_0 is the concentration at the point in the axial flow into which the feed is injected ($C_0 \neq C_F$, generally speaking), C_p is the product concentration and C_w is the waste concentration.

From Eqs. (2.8)–(2.10), the variables E_F and m in Eq. (2.7) in the enriching and stripping sections are regarded as constants respectively, and then the boundary concentration ratios of the enriching and stripping sections are obtained as follows:

$$\frac{(C_p - b_e)(C_0 - a_e)}{(C_p - a_e)(C_0 - b_e)} = e^{k_{0e}(a_e - b_e)(\eta_H - \eta_F)} \tag{2.11}$$

$$\frac{(C_w - b_s)(C_0 - a_s)}{(C_w - a_s)(C_0 - b_s)} = e^{-k_{0s}(a_s - b_s)\eta_F} \tag{2.12}$$

Where, subscript e indicates the enriching section and s indicates the stripping section. From Eqs. (2.11) and (2.12), the product concentration and the waste concentration are solved as

$$C_P = \frac{b_e + a_e((C_0 - b_e)/(a_e - C_0))e^{k_{0e}(a_e - b_e)(\eta_H - \eta_F)}}{1 + ((C_0 - b_e)/(a_e - C_0))e^{k_{0e}(a_e - b_e)(\eta_H - \eta_F)}} \tag{2.13}$$

$$C_W = \frac{a_s + b_s((a_s - C_0)/(C_0 - b_s))e^{k_{0s}(a_s - b_s)\eta_F}}{1 + ((a_s - C_0)/(C_0 - b_s))e^{k_{0s}(a_s - b_s)\eta_F}} \tag{2.14}$$

In a steady state, the balance equation on the net flow flux in a centrifuge is

$$C_F = \theta C_P + (1 - \theta)C_W \tag{2.15}$$

where θ is the cut and C_F is the feed concentration.

From Eqs. (2.13)–(2.15), the solution of the head and tail concentration, that is, C_p and C_w, is obtained with the operation parameters given in a gas centrifuge. The overall separation factor for the unit molar weight difference, γ_0 can finally be obtained:

$$\gamma_0 = \frac{C_P/(1 - C_P)}{C_W/(1 - C_W)} \tag{2.16}$$

Equations (2.13) and (2.14) are transcendental equations, as the coefficients $a_{e,s}$ and $b_{e,s}$ contain the withdrawal concentration (C_p and C_w). The equations are solved with a numerical method. The value of γ_0 solved by Eqs. (2.13)–(2.16) can be used with an arbitrary magnitude of concentration.

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A PURELY AXIAL FLOW ANALYTICAL SOLUTION FOR ARBITRARY A^2

To solve Eqs (2.13)–(2.16), the velocity distribution in a gas centrifuge must be known, i.e., the V_z or ψ. The velocity field can be obtained in a variety of ways, such as with the Onsager pancake model reported by Wood and Morton (8). However, here a simplified model is used for the purpose of illustration. The model is for a purely axial flow in a gas centrifuge, that is, the influence of ends of a centrifuge on internal flow is ignored and the internal flow remains only with an axial direction velocity (4–7). In this paper the analytical solution of the model does not have to be subject to the condition that dimensionless speed parameter $A^2 \gg 1$, but an arbitrary magnitude can be given for A^2.

Eq. (3.1) is obtained by eliminating the perturbation density and pressure terms through simplifying the continuity equation, the equations of motion in the radial and axial directions, and the state equation according to the purely axial flow assumption, that is, $V_r = 0$, $V_\theta = 0$, $\partial V_z/\partial z = 0$. Eq (3.2) correspond to the energy equation (ref. 7, or ref. 8).

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$$\mu \frac{\partial}{\partial r}\left(\frac{\partial^2 V_z'}{\partial r^2} + \frac{1}{r}\frac{\partial V_z'}{\partial r}\right) = \frac{\mu M \Omega^2 r}{RT_0}\left(\frac{\partial^2 V_z'}{\partial r^2} + \frac{1}{r}\frac{\partial V_z'}{\partial r}\right) - \frac{\rho_0 \Omega^2 r}{T_0}\frac{\partial T'}{\partial z} \tag{3.1}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right) + \frac{\partial^2 T'}{\partial z^2} \tag{3.2}$$

Where, μ is the gas viscosity, Ω is the angular velocity of the cylinder, V_z, V_r and V_θ are the components of velocity in the axial, radial and azimuthal directions, ρ and T denote the thermodynamic variable mass density and temperature. The subscript 0 indicates the isothermal solid body rotation solution, and the superscript $'$ indicates the perturbations which are the deviations of the isothermal solid body rotation solution.

The following dimensionless variables are defined [similar to (9)] as:

$$\begin{aligned} &\mathrm{R_e} \equiv \frac{\rho_w \Omega r_a^2}{\mu} = \frac{M p_w \Omega r_a^2}{\mu R T_0}; \quad \xi \equiv A^2\left(1 - \frac{r^2}{r_a^2}\right); \quad s \equiv \frac{z}{r_a}; \\ &w \equiv \frac{V_z'}{\Omega r_a}; \quad t \equiv \frac{T'}{T_0} \end{aligned} \tag{3.3}$$

In terms of the dimensionless variables above, the purely axial flow conditions indicate $V_r' = 0$, $V_\theta' = 0$. V_z' is only a function of radial coordinate

($d/d\,\xi$ instead of $\partial/\partial\,\xi$), and Eqs. (3.1) and (3.2) can be rewritten as

$$\frac{d^2}{d\xi^2}\left[\left(1-\frac{\xi}{A^2}\right)\frac{dw}{d\xi}\right]+\frac{d}{d\xi}\left[\left(1-\frac{\xi}{A^2}\right)\frac{dw}{d\xi}\right]=\frac{\mathrm{R_e}}{8A^6}e^{-\xi}\frac{\partial t}{\partial s} \quad (3.4)$$

$$4A^4\frac{\partial}{\partial\xi}\left[\left(1-\frac{\xi}{A^2}\right)\frac{\partial t}{\partial\xi}\right]+\frac{\partial^2 t}{\partial s^2}=0 \quad (3.5)$$

According to the purely axial flow assumption, the axial velocity w is independent of the radial coordinate. So the temperature field should be a function of ξ in the right-hand side of the Eq. (3.4), $\partial t/\partial s = f(\xi)$. So $\partial^2 t/\partial s^2 = 0$. After integrating Eq. (3.5), the temperature field becomes $t = f_1(s)\ln(1-\xi/A^2)+f_2(s)$. The temperature is finite on the axis ($\xi = A^2$), so $f_1(s) = 0$. Let the temperature field be $t = h\cdot s + B$. Here, h and B are undetermined constant. The temperature gradient is determined by the wall temperature, $h = dt/ds = (r_a/T_0)(dT_w/dz)$, where T_w is the wall temperature. To reduce Eqs. (3.4) and (3.5) further, a three-order differential equation is obtained finally as follows

$$\frac{d^2}{d\xi^2}\left[\left(1-\frac{\xi}{A^2}\right)\frac{dw}{d\xi}\right]+\frac{d}{d\xi}\left[\left(1-\frac{\xi}{A^2}\right)\frac{dw}{d\xi}\right]=\lambda e^{-\xi} \quad (3.6)$$

where $\lambda = \mathrm{R_e}\,h/(8A^6)$. The following boundary conditions are used for finding three integral constants:

1) No slip on the wall condition $\xi = 0 \quad w = 0$;

2) Symmetry condition $\xi = A^2\cdot\varepsilon \;\; \dfrac{dw}{d\xi} = 0$ (3.7)

3) Feed condition $P^* = \displaystyle\int_{r_\varepsilon}^{r_a}\rho_0\overline{V}_z 2\pi r dr = \frac{\pi\rho_w\Omega r_a^3}{A^2}\int_0^{A^2\varepsilon} we^{-\xi}d\xi$

where P^* is the net axial flow flux of the gas, $P^* = 0$ (in the total reflux case); $P^* = P$ (the enriching section), $P^* = -W$ (the stripping section). In specifying the boundary conditions, since Eq (3.6) degenerates at $\xi = A^2$, condition 2) is given at $\xi = A^2\,\varepsilon$, with $\varepsilon < 1$. Likewise, in order to avoid the pole, the integration in condition 3) is carried out over an interval $[0, A^2\varepsilon]$, slightly smaller than $[0, A^2]$. After the equation is solved, we let $\varepsilon \to 1$.

Integrating Eq. (3.6) three times with respect to ξ, the dimensionless axial velocity is obtained:

$$w = B_1 e^{-\xi} + B_2 E_i\left[A^2\left(1-\frac{\xi}{A^2}\right)\right] + B_3\ln\left(1-\frac{\xi}{A^2}\right) + B_4 \quad (3.8)$$

where $E_i(x)$, called the exponential integral function, is defined by $E_i\,(x) \equiv -\int_{-x}^{\infty} e^{-t}/t\;dt$ $(x > 0)$, where the principal value of the integral is taken. The coefficients B_1, B_2, B_3, and B_4 are independent of the radial coordinate

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ξ, and C_1, C_2, and C_3 are integral constants:

$$B_1 = -A^2\lambda; \quad B_2 = A^2 e^{-A^2}(A^2\lambda - C_2); \quad B_3 = -A^2 C_1; \quad B_4 = C_3$$

$$C_1 = e^{-A^2\varepsilon}(A^2\varepsilon\lambda - C_2)$$

$$C_3 = A^2[\lambda + e^{-A^2}E_i(A^2)(C_2 - A^2\lambda)]$$

$$C_2 = e^{2A^2}[A^2\lambda - 2A^2 e^{A^2\varepsilon}\lambda + e^{2A^2\varepsilon}(-2Q + A^2\lambda)] + 2A^4\lambda\{-e^{2A^2\varepsilon}E_i(2A^2) + e^{2A^2\varepsilon}E_i(-2A^2(\varepsilon - 1)) + e^{A^2(1+\varepsilon)}(1+\varepsilon)[E_i(A^2) - E_i(-A^2(\varepsilon - 1))] + e^{2A^2}\varepsilon\ln(1-\varepsilon)\}/2A^2\{-e^{2A^2\varepsilon}E_i(2A^2) + e^{2A^2\varepsilon}E_i(-2A^2(\varepsilon - 1)) + 2e^{A^2(1+\varepsilon)}[E_i(A^2) - E_i(-A^2(\varepsilon-1))] + e^{2A^2}\ln(1-\varepsilon)\}$$

Here $Q = P^* A^2/(\pi\rho_w\Omega r_a^3)$.
Then the axial velocity of the purely axial flow is

$$V_z = \Omega r_a\left\{B_1 e^{-\xi} + B_2 E_i\left[A^2\left(1 - \frac{\xi}{A^2}\right)\right] + B_3\ln\left(1 - \frac{\xi}{A^2}\right) + B_4\right\} \quad (3.9)$$

The stream function with ξ is expressed as

$$\psi = \frac{2\pi p_w r_a}{\Omega}\int_{\xi}^{A^2\varepsilon} e^{-\xi}w(z)d\xi \quad (3.10)$$

Substituting (3.8) into (3.10), the stream function is obtained as follows

$$\psi = \frac{2\pi p_w r_a}{\Omega}\left\{\frac{1}{2}B_1(e^{-2\xi} - e^{-2A^2\varepsilon}) + B_2[e^{-A^2}E_i(2A^2 - 2A^2\varepsilon) - e^{-A^2\varepsilon}E_i(A^2 - A^2\varepsilon) - e^{-A^2}E_i(2A^2 - 2\xi) + e^{-\xi}E_i(A^2 - \xi)] + B_3\left[e^{-A^2}E_i(A^2 - A^2\varepsilon) - e^{-A^2}E_i(A^2 - \xi) - e^{-A^2\varepsilon}\ln(1-\varepsilon) + e^{-\xi}\ln\left(1 - \frac{\xi}{A^2}\right)\right] + B_4(e^{-\xi} - e^{-A^2\varepsilon})\right\} \quad (3.11)$$

The flow pattern efficiency, which is related to the distribution of the mass flux ρV_z along the radius and evaluates the effect of the distribution of the mass flux on the separation power, is defined as

$$E_F \equiv \frac{\left(2\int_{r_\varepsilon}^{r_a}\psi r dr\right)^2}{r_a^4\int_{r_\varepsilon}^{r_a}(\psi^2/r)dr} \quad (3.12)$$

where r_ε corresponds to the radius when $\xi = A^2\varepsilon$, i.e., $r_\varepsilon = \sqrt{1-\varepsilon}\,r_a$. Substituting (3.11) into (3.12), the analytical solution of the flow pattern efficiency can be obtained. The solution expression is more complicated. The solution has greater application range than Von Halle (7).

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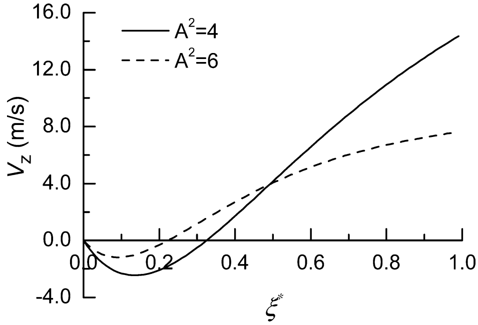

Figure 1. Profile of the axial velocity in total reflux case.

RESULTS AND DISCUSSION

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The overall separation factor for the unit molar weight mass difference, γ_0, is an important characteristic separation parameter and depends on many parameters. Note that from Eq. (2.7) γ_0 is a function of E_F, whereas E_F is related to the temperature gradient on the wall. In this paper, the value of γ_0 is obtained by optimization of the temperature gradient on the wall in all cases of calculations. In our calculation one parameter is changed and all the other parameters are unchanged. The feed concentration has little influence on γ_0 (3), so the feed concentration is supposed as a fixed value.

In the separation theory, the dimensionless speed parameter A^2 is defined by $A^2 \equiv M(\Omega\, r_a)^2/(2RT_0)$. The parameter A^2 plays a very important role in the separation performance in a gas centrifuge. From the definition of A^2, the principally influential parameters on A^2 are the gas molecular weight M and the linear speed Ωr_a. In this paper, we consider that the change of A^2 resulted only from changes in the molecular weight M.

The value of the temperature gradient on the wall in the flow field calculations is optimized in a total reflux case ($F = 0$). The temperature gradient is

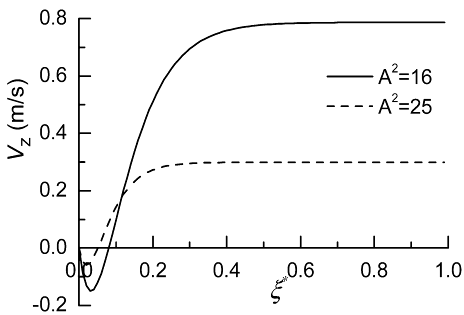

Figure 2. Profile of the axial velocity in total reflux case.

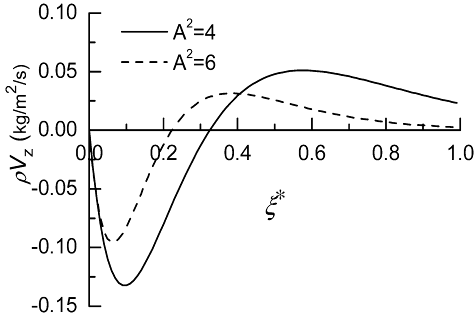

Figure 3. Profile of the axial mass flux in total reflux case.

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taken as 0.5 K/m in a withdrawal case ($F \neq 0$). It is convenient to convert the radial coordinate by $\xi^* = \xi/A^2$ in the following figures. The following parameters are fixed for all calculations: Length-diameter ratio $z_H/r_a = 30$. The axial location of the feed point is the axial length ratio $z_F/z_H = 0.5$. The linear speed $V = \Omega r_a = 500\,\mathrm{m/s}$. The average temperature of gas $T_0 = 300\,\mathrm{K}$. The gas viscosity $\mu_0 = 1.746 \times 10^{-5}\,\mathrm{kg/m/s}$. The cut $\theta = 0.45$. The pressure of the process gas at the rotor wall $p_w = 20\,\mathrm{mmHg}$. The feed rate of the process gas $F = 80\,\mathrm{g/h}$. The product $\rho D_0 = 2.297 \times 10^{-5}\,\mathrm{kg/m/s}$.

The results of calculation are shown in Figs. 1 through 10. The distributions of the axial velocity, the axial mass flow and the axial flow pattern efficiency for different the values of A^2 in a total reflux case are shown in Figs. 1 to 5. The curves show that the profiles are different for different values of A^2. In Fig. 1, it seems that the boundary condition $\partial w/\partial\xi = 0$ is not satisfied at the axis. But the original physical boundary condition $\partial w/\partial r = 0$ at the axis, since $\partial w/\partial r = -2A^2\, r\partial w/\partial\xi = 0$ at $r = 0$ because, clearly, $\partial w/$

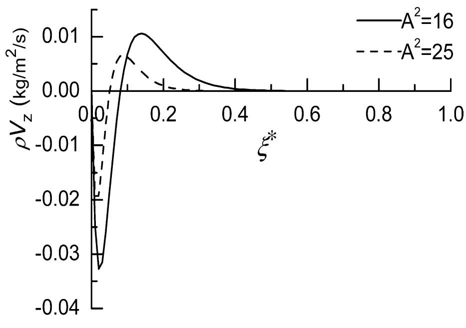

Figure 4. Profile of the axial mass flux in total reflux case.

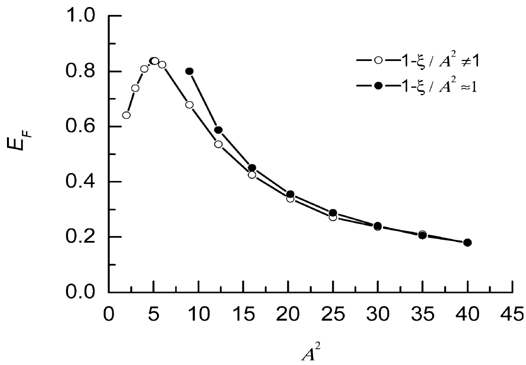

Figure 5. Profile of the flow pattern efficiency in total reflux case.

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$\partial\xi$ is finite according to Fig. 1. Figure 3 shows that even in a higher linear speed centrifuge, the mass flux of the process gas with smaller A^2 is not mainly in the thin layer approaching the wall of the rotor. Figures 2 and 4 show that the distribution tendency of larger values of A^2 agrees with Von Halle (7). The curves in Fig. 5 are the profiles of the flow pattern efficiency with A^2 varying in a total reflux case. The flow pattern efficiency reaches its maximum value when A^2 is between 5 and 6. The tendency of the flow pattern efficiency with varied A^2 found in this study agrees with the result of the thin layer approximation reported by Von Halle ($1 - \xi/A^2 \approx 1$, $E_F = 7.2/A^2$) (7) when the value of A^2 exceeds 7.2. Figures 1 through 4 show that the zero location point, where the axial velocity changes direction, moves to the wall of the rotor with the variation of A^2. A calculation for a withdrawal case was also performed. The curves of the axial velocity, the

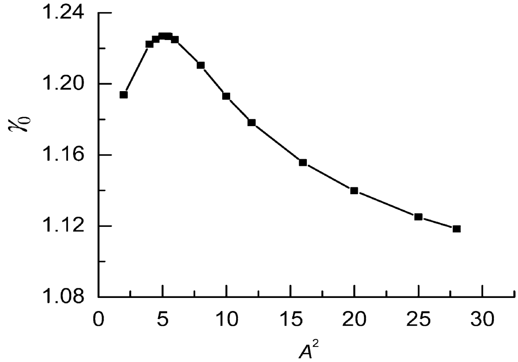

Figure 6. Dependence of γ_0 on A^2.

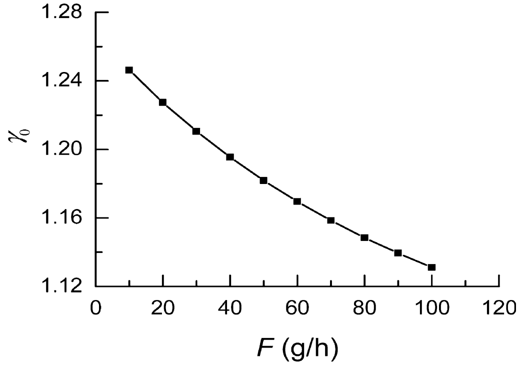

Figure 7. Dependence of γ_0 on F.

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mass flux and the flow pattern efficiency in the withdrawal case were all similar to a total reflux case. This is not surprising, as the flow model takes into account the thermal counter-current only.

Figures 6 through 10 show the dependence of γ_0 on the speed parameter A^2 ($A^2 = 15$ in Figs. 7 through 10, the feed flow rate F, the product ρD (the value of ρD is scaled by ρD_0 which is a suppositional one in calculation), the wall pressure p_w, and the cut θ. As mentioned above, the value of A^2 is changed only when the molecular weight M is changed. In Figure 6, γ_0 reaches its maximum when A^2 is between 5 and 6. The reason is that the flow pattern efficiency has a maximum at $A^2 = 5$–6 (see Fig. 5). Figures 7 and 8 show that γ_0 decreases with the feed rate F and increases with the product ρD. The curves of γ_0 vs. p_w for different F are shown in Fig. 9. The γ_0 has its maximum with p_w and its extremum diminishes with the feed rate F. The dependence of γ_0 on θ in Fig. 10 shows that γ_0 changes a little with the minimum at $\theta \approx 0.5$. The dependence of γ_0 on the speed parameter A^2,

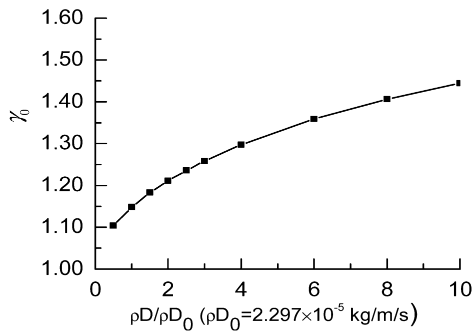

Figure 8. Dependence of γ_0 on ρD.

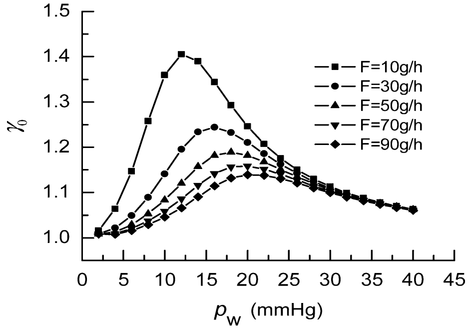

Figure 9. Dependence of γ_0 on P^w.

the feed flow rate F, the product ρD and the cut θ agrees well with the results reported by Wood et al. (3).

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CONCLUSIONS

The analytical solution of the purely axial flow field and the overall separation factor, γ_0, for a binary isotope mixture in a gas centrifuge were solved in this paper. From the previous discussion we can obtain the following conclusions:

A) An analytical solution of the purely axial flow in a gas centrifuge was derived and solved by reducing the linearized motion equations of a gas centrifuge. The solution was applied to an arbitrary value of A^2.

B) The calculation examples illustrate that the mass flux of the smaller A^2 does not exist mainly in the thin layer near the rotor wall but also at the axis even in a high-rotation-speed centrifuge. A^2 can be small when the

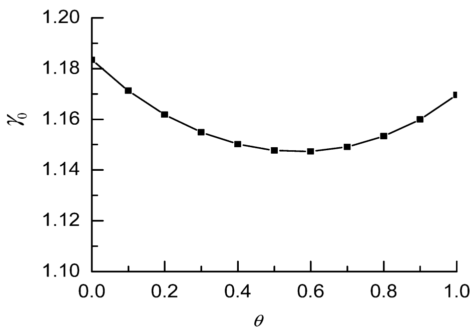

Figure 10. Dependence of γ_0 on θ.

process gas is light. The flow pattern efficiency reaches its maximum when A^2 is 5–6.

C) The expression of γ_0 for a binary isotope mixture is obtained by solving the unit molar weight mass difference non-linear diffusion equation. There is no limit to the range of concentration value. The expression of γ_0 is suitable for arbitrary concentration.

D) From the example, the speed parameter A^2, the product ρD, the molecular weight of process gas M, the pressure of the wall p_w and the feed rate F have great influence on the value of γ_0. γ_0 changed a little with a minimum at $\theta \approx 0.5$. The separation factor γ_0 reaches its maximum when the speed parameter A^2 is 5–6.

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